4. L. N. Maurin, "Equilibrium of a liquid film on a rotating sphere," Prikl. Mat. Mekh., 33, No. 1 (1969).
5. Rayleigh, Theory of Sound [Russian translation], Vol. 2, OGIZ, Moscow–Leningrad (1944).

STRUCTURE OF COMPLETELY DISPERSED SHOCK WAVES IN RELAXING MIXTURES

A. L. Ni and O. S. Ryzhov

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Shock waves in chemically active gas mixtures with an arbitrary number of reactions are discussed. It is assumed that the difference between the frozen and equilibrium sound velocities calculated from the unperturbed state of the material is a small quantity relative to one of these velocities. The flow rate at infinity is assumed to lie within the range between the frozen and equilibrium sound velocities; the shock wave does not then contain discontinuities, i.e., it possesses complete dispersion. Different cases which may be encountered upon increasing the velocity of the advancing flow are successively investigated. The method of splicing exterior and interior asymptotic expansions is used to construct a solution.

1. Formulation of the Problem. Let us apply to the investigation of the structure of weak shock waves in multicomponent relaxing media the system of equations which describes one-dimensional steady flow in the transonic velocity range

$$2\left(\varepsilon m_\infty v' + \varepsilon_a^2 \gamma_f\right)\frac{dv'}{dx'} = \delta_a^2 \mathbf{e}_2' \frac{d\mathbf{q}_2'}{dx'},\ \ \delta_a^2 = \frac{p_\infty}{\rho_\infty v_\infty^2}\varepsilon_a^2, \qquad (1.1)$$

$$\frac{d\mathbf{q}_2'}{dx'} = -\mathbf{E}\boldsymbol{\omega}_2',\ \ \boldsymbol{\omega}_2' = \mathbf{D}\mathbf{q}_2' + \mathbf{e}_2' v'.$$

Both the length along the coordinate x' and the velocity v' of the perturbed motion of particles together with the components of the vectors $\mathbf{q}_2' = (q_{21}', \ldots, q_{2N}')$ and $\boldsymbol{\omega}_2' = (\omega_{21}', \ldots, \omega_{2N}')$ of the completeness and affinity of the chemical reactions are taken here in a special dimensionless system of units. The density, pressure, and dimensionless thermodynamic coefficient, which is proportional to the curvature of the Poisson adiabat for a mixture with constant composition, are denoted by the letters ρ, p, and m, respectively. The subscript ∞ refers to the state of the material in the advancing uniform flow. The small parameter ε is proportional to the amplitude of the perturbations, and the appearance of the other small parameter ε_a^2 is dictated by the conditions for providing closeness of the frozen $a_{f\infty}$ and the equilibrium $a_{e\infty}$ sound velocities in the unperturbed state. It is assumed in the derivation of Eqs. (1.1) that the velocity v_∞ of the advancing flow deviates slightly from both the frozen and equilibrium sound velocities; the number γ_f is used to specify this deviation, to wit,

$$v_\infty - a_{f\infty} = \varepsilon_a^2 \gamma_f v_\infty. \qquad (1.2)$$

Any two positive-definite and symmetrical matrices can appear in the original Euler equations as the kinetic matrix and the stability matrix of the system. Linear transformations of the completeness and affinity vectors of the chemical reactions permit reducing these matrices to the unit E and diagonal D matrices, respectively. This transformation is assumed to be carried out in the system of Eqs. (1.1). The components of the constant dimensionless vector $\mathbf{e}_2' = (e_{21}', \ldots, e_{2N}')$, which are proportional to the adiabatic derivatives of the specific internal energy of the system with respect to the specific volume and one of the components of the completeness vector of the reactions, are also assumed to be subject to the indicated linear transformations.

Since the advancing flow is uniform and is in a state of complete thermodynamic equilibrium,

$$v' \to 0,\ \mathbf{q}_2' \to 0,\ \frac{dv'}{dx'} \to 0,\ \frac{d\mathbf{q}_2}{dx'} \to 0 \quad \text{as} \quad x' \to -\infty. \qquad (1.3)$$

The gas mixture reaches a new equilibrium state as a result of compression within the shock wave; therefore,

$$v' \to v_0',\ \frac{dv'}{dx'} \to 0,\ \frac{d\mathbf{q}_2'}{dx'} \to 0 \quad \text{as} \quad x' \to +\infty. \qquad (1.4)$$

The boundary conditions (1.3) and (1.4) determine the solution with an accuracy to an insignificant shift in x'.

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As will be evident in what follows, sometimes it is more convenient to operate with an equation of the (N+1)-th order for the perturbed velocity

$$\sum_{k=0}^{N} \sigma_{N-k} \frac{d^k}{dx'^k}\left[\left(\varepsilon m_\infty v' + \varepsilon_a^2 \gamma^{(k)}\right) \frac{dv'}{dx'}\right] = 0 \tag{1.5}$$

instead of (1.1) [1]. Here the symbol σ_l denotes the sum of all possible products which are composed of the positive eigenvalues $\lambda_1, \ldots, \lambda_N$, which are equal, respectively, to the diagonal elements $d_{11}, \ldots, d_{NN}$ of the relaxation matrix $R = ED = D$ and which are taken l at a time in each product. The parameters $\gamma^{(k)}$ characterize the deviations of each of the so-called intermediate sound velocities $\alpha_{k\infty}$ from the velocity of the advancing flow, namely,

$$v_\infty - \alpha_{k\infty} = \varepsilon_a^2 \gamma^{(k)} v_\infty. \tag{1.6}$$

The intermediate sound velocities themselves are expressed by the formulas

$$\alpha_{k\infty} = a_{f\infty} + \frac{1}{2} \delta_a^2 v_\infty \sum_{m=k+1}^{N} (-1)^{m-k} \frac{\sigma_{N-m}}{\sigma_{N-k}} \mathbf{e}_2' \mathbf{D}^{m-k-1} \mathbf{e}_2' \tag{1.7}$$

and are subject to the inequalities [2-4]

$$a_{e\infty} = \alpha_{0,\infty} < \alpha_{1,\infty} < \ldots < \alpha_{N-1,\infty} < \alpha_{N,\infty} = a_{f\infty}. \tag{1.8}$$

In the limiting cases k = 0 and k = N we obtain $\gamma^{(0)} = \gamma_e$ and $\gamma^{(N)} = \gamma_f$. Thus Eq. (1.6) changes into (1.2) when the intermediate sound velocity coincides with the propagation velocity of the perturbations in a mixture with a constant chemical composition.

Let us integrate the first of Eqs. (1.1). Having determined the arbitrary constant from the boundary conditions as $x' \to -\infty$, we have

$$\varepsilon m_\infty \left(v' + \frac{\varepsilon_a^2 \gamma_f}{\varepsilon m_\infty}\right)^2 - \delta_a^2 \mathbf{e}_2' \mathbf{q}_2' = \frac{\varepsilon_a^4 \gamma_f^2}{\varepsilon m_\infty}. \tag{1.9}$$

Having made use of the boundary conditions as $x' \to +\infty$ and the relation

$$a_{e\infty} = a_{f\infty} - \frac{1}{2} \delta_a^2 v_\infty \sum_{i=1}^{N} \frac{e_{2i}'^2}{\lambda_i} \tag{1.10}$$

between the frozen and equilibrium sound velocities, which follows from (1.7) when k = 0, we find

$$v_0' = -\frac{2\varepsilon_a^2 \gamma_e}{\varepsilon m_\infty}.$$

The latter equation offers the possibility of specifying the coefficients γ_e or γ_f instead of the constant v_0'.

For the sake of brevity, in the following we will omit in the symbols of both the constants and the variable quantities the primes and the subscript 2, which indicates the result of a linear transformation of the completeness and affinity vectors of the chemical reactions.

2. General Properties of the Solution. The problem of the internal structure of shock waves is among the classical ones. A comprehensive review of the theoretical and experimental investigations carried out up to 1965 which are devoted to this problem is contained in [5, 6]. The principal conclusion of the early papers reduces to the fact that there exists a sequence of relaxation zones situated next to one another if the rates of the chemical reactions differ appreciably from each other. These zones have various widths determined by one or several relaxation processes. One of the first attempts to give a quantitative analysis of the phenomenon was evidently undertaken in [7]; numerical calculations confirmed the conclusion drawn on the basis of simple physical considerations that shock waves have a "banded" structure. The problem of transient perturbations excited in a gas mixture by the motion of a flat piston has been recently solved [8]. The data obtained show how the stratification in time of relaxation zones with different widths occurs.

The solutions of the nonlinear system of equations (1.1) possess all the qualitative characteristics which the solutions constructed in [7] with the help of numerical methods do. These equations permit a systematic analysis based on the method of splicing exterior and interior asymptotic expansions. Moreover, the assumption of the closeness of the frozen and equilibrium sound velocities permits investigating all the regularities inherent to the "banded" structure of a shock wave. We note that the assumption indicated does not lead to great restrictions, since the relative difference between the frozen and equilibrium sound velocities lies within the 10% range for many actual chemically active gas mixtures.

In order to exclude the formation of discontinuities in the shock wave, modes with $a_{f\infty} > v_\infty > a_{e\infty}$ will be investigated below. Setting k = 0 and k = N in Eq. (1.6), we have

$$\gamma_f < 0 < \gamma_e. \tag{2.1}$$

When $\gamma_f > 0$, the advancing flow is subjected at first to an abrupt compression, and then its parameters vary continuously.

It is simplest to find the asymptotic form of the solution of the system of equations (1.1) as $x \to -\infty$ if one neglects the quadratic term in the integral (1.9). Finally,

$$2\gamma_f v = \frac{p_\infty}{\rho_\infty v_\infty^2} \mathbf{e}\cdot\mathbf{q}.$$

From this follows the equation

$$\frac{d\mathbf{q}}{dx} = -\left(\mathbf{D} + \frac{1}{2}\frac{p_\infty}{\rho_\infty v_\infty^2 \gamma_f}\mathbf{B}\right)\mathbf{q} \tag{2.2}$$

for the completeness vector of the chemical reactions, where **B** is a symmetric matrix with elements $b_{ik} = e_i e_k$.

We will investigate what the eigenvalues of the symmetric matrix

$$\mathbf{S} = \mathbf{D} + \frac{1}{2}\frac{p_\infty}{\rho_\infty v_\infty^2 \gamma_f}\mathbf{B}, \tag{2.3}$$

are, whose elements are s_{ik}. To this end we consider the quadratic form

$$\varphi(\xi_i, \xi_k) = \sum_{i,k=1}^{N} s_{ik}\xi_i\xi_k = \sum_{i=1}^{N}\lambda_i\xi_i^2 + \frac{1}{2}\frac{p_\infty}{\rho_\infty v_\infty^2 \gamma_f}\left(\sum_{i=1}^{N} e_i\xi_i\right)^2. \tag{2.4}$$

We introduce the linear transformation

$$\xi = \mathbf{C}\eta,$$

which is specified by the relations

$$\eta_i = \xi_i,\ i = 1, \ldots, N-1;\ \eta_N = \sum_{i=1}^{N} e_i\xi_i.$$

In the new variables the quadratic form (2.4) takes the form

$$\varphi(\xi_i, \xi_k) = \psi(\eta_i, \eta_k) = \sum_{i,k=1}^{N} w_{ik}\eta_i\eta_k,$$

and the matrices S and $\mathbf{W} = \|W_{ik}\|$ are related by the equation

$$\mathbf{W} = \mathbf{C}^*\mathbf{S}\mathbf{C} = \mathbf{C}^*\mathbf{D}\mathbf{C} + \frac{1}{2}\frac{p_\infty}{\rho_\infty v_\infty^2 \gamma_f}\mathbf{C}^*\mathbf{B}\mathbf{C}. \tag{2.5}$$

Here the matrix which is the transpose to **C** is denoted, as usual, by the symbol $\mathbf{C}^*$.

The matrix $\mathbf{D}^{(1)} = \mathbf{C}^*\mathbf{D}\mathbf{C}$ possesses a rather complex structure; however, it is possible to assert that it is symmetric and positive-definite by virtue of the law of inertia of quadratic forms. Concerning the matrix $\mathbf{B}^{(1)} = \mathbf{C}^*\mathbf{B}\mathbf{C}$, it has the single nonzero element $b_{NN}^{(1)} = 1$, as direct calculation shows. It follows from this that the matrix **W** is symmetric, and N − 1 of its principal minors

$$\Delta_1 = w_{11},\ \Delta_2 = \begin{vmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{vmatrix}, \ldots, \Delta_{N-1} = \begin{vmatrix} w_{11} & \ldots w_{N-1,1} \\ \vdots & \vdots \\ w_{1,N-1} & \ldots w_{N-1,N-1} \end{vmatrix}$$

coincide with the analogous minors of the matrix $\mathbf{D}^{(1)}$. Since the latter is positive-definite, we conclude on the basis of the Sylvester criterion [9] that $\Delta_1, \Delta_2, \ldots, \Delta_{N-1}$ are positive.

It remains to investigate the sign of the N-th principal minor Δ_N, which is simply det **W**. By virtue of the properties of the determinants of the product of matrices, we have

$$\det \mathbf{W} = \det \mathbf{C}^* \cdot \det \mathbf{S} \cdot \det \mathbf{C} = (\det \mathbf{C})^2 \cdot \det \mathbf{S},$$

i.e., the signs of det **W** and det **S** are identical. As a result of simple calculations we find

$$\det \mathbf{S} = \prod_{i=1}^{N} \lambda_i + \frac{1}{2} \frac{p_\infty}{\rho_\infty v_\infty^2 \gamma_f} \sum_{j=1}^{N} e_j^2 \prod_{i=1}^{N\,(j)} \lambda_i, \tag{2.6}$$

where the superscript (j) next to the product $\prod\limits_{i=1}^{N(j)} \lambda_i$ indicates that λ_j is excluded from the complete set of its cofactors λ_i. Subsequent transformation of (2.6) leads to

$$\det \mathbf{S} = \prod_{i=1}^{N} \lambda_i \left[1 + \frac{1}{2} \frac{p_\infty}{\rho_\infty v_\infty^2 \gamma_f} \sum_{j=1}^{N} \frac{e_j^2}{\lambda_j} \right].$$

It is evident from Eq. (1.10) that the expression in square brackets on the right-hand side of the last equation is written as $1 + (\gamma_e - \gamma_f)/\gamma_f$. Since all the eigenvalues λ_i ($i = 1, \ldots, N$) are positive, the condition $\det \mathbf{S} < 0$ follows from the inequalities (2.1). Thus the N-th principal minor Δ_N of the matrix **W**, which was introduced by Eq. (2.5), has a negative sign.

Now let us apply the Jacobi method for the reduction of a quadratic form $\psi(\eta_i, \eta_k)$ to a sum of squares [9]. This method permits writing the coefficient of the j-th term in the form of the ratio Δ_{j-1}/Δ_j. Thence it is clear that $N - 1$ eigenvalues of the symmetric matrix **W** are positive, and one eigenvalue is negative. By virtue of the law of inertia of quadratic forms one can confirm that the eigenvalues of the original matrix S defined by Eq. (2.3) possess the analogous property. Let us denote the eigenvalues of this matrix by l_i and set $l_1, \ldots, l_{j-1} > 0$, $l_j < 0$, $l_{j+1}, \ldots$, and $l_N > 0$.

Let us return to Eq. (2.2). The substitution into it of the expression

$$\mathbf{q} = \mathbf{q}_0 \, e^{\mu x} \tag{2.7}$$

for the completeness vector of the chemical reactions gives

$$(\mathbf{S} + \mu \mathbf{E}) \mathbf{q}_0 = 0. \tag{2.8}$$

Since the condition $\mathbf{q} \to 0$ as $x \to -\infty$ occurs, the integrals of Eq. (2.2) with $\mu = -l_i$ ($i = 1, \ldots, j-1, j+1, \ldots, N$) should be discarded. From this we conclude that the coefficient in the exponent in Eq. (2.7) takes the single value $\mu = -l_j$. The homogeneous equation (2.8) determines the vector $\mathbf{q}_0$ with an accuracy of an arbitrary factor. Bearing in mind the relation between the perturbed velocity of the particles and the completeness vector of the chemical reactions, let us represent the asymptote of the solution as $x \to -\infty$ in the form

$$v = c e^{\mu x}, \quad q_i = -\frac{e_i c}{\mu + \lambda_i} e^{\mu x}, \quad \mu = -l_j \quad (i = 1, \ldots, N). \tag{2.9}$$

We will now prove that the thermodynamic parameters q_i increase monotonically, and the velocity of the gas falls off monotonically along the coordinate. Let us introduce for the stated purpose the auxiliary quantities $q_i^x = \lambda_i q_i / e_i$. Treating v as a known function, we have

$$q_i^x = -\lambda_i \int_{-\infty}^{x} v(\xi) \, e^{\lambda_i(\xi - x)} d\xi \quad (i = 1, \ldots, N). \tag{2.10}$$

One can verify that the integral forms for q_i^x agree with the asymptotic Eqs. (2.9) as $x \to -\infty$. By virtue of the first of the indicated equations there exists a range $-\infty < x < x_0$ in which the derivative $dv/dx < 0$ if the arbitrary constant $c < 0$. Solutions with $c > 0$ must be excluded from consideration, since they do not satisfy the boundary conditions (1.4) as $x \to \infty$. A monotonic nature of the variation of v will be established if it turns out that $dv(x_0)/dx < 0$.

Let us assume the opposite and set $dv(x_0)/dx = 0$. Then in accordance with the first equation of the system (1.1) we have

$$\sum_{i=1}^{N} \frac{e_i^2}{\lambda_i} \frac{dq_i^x(x_0)}{dx} = 0 \tag{2.11}$$

at the point x_0. The combination of the remaining equations of this system with Eqs. (2.10) gives

$$\frac{dq_i^x(x_0)}{dx} = -\lambda_i^2 \int_{-\infty}^{x_0} [v(x_0) - v(\xi)] \, e^{\lambda_i(\xi - x_0)} d\xi \quad (i = 1, \ldots, N). \tag{2.12}$$

The function v(x) reaches its minimum value $v(x_0)$ on the edge of the interval $-\infty < x \le x_0$. From this it follows that the integrand in (2.12) is negative, and the derivatives

$$\frac{dq_i^x(x_0)}{dx}>0\ (i=1,\ldots,N).$$

Summing the last inequalities multiplied in advance by e_i^2/λ_i, we arrive at a contradiction to Eq. (2.11). This contradiction proves that the gas velocity falls off monotonically along the coordinate x. Appeal to Eqs. (2.12) leads to the conclusion of a monotonic increase of the components of the completeness vector of the chemical reactions.

3. Roots of the Characteristic Equation. The eigenvalues of the matrix S are determined as the roots of the characteristic equation which follows from (2.8) taken with opposite sign. It is obtained in most compact form if one substitutes the first of Eqs. (2.9) into Eq. (1.5) for the perturbed velocity and discards the lowest terms in the equation thus found. As a result,

$$\sum_{k=0}^{N}\sigma_{N-k}\gamma^{(k)}\mu^{k}=P_N=0. \tag{3.1}$$

As has been proven above, Eq. (3.1) has N real roots $\mu_1=-l_1, \ldots, \mu_N=-l_N$; only one of them is positive, and the rest are negative. Let us arrange these roots in the following order:

$$|\mu_i|\geqslant|\mu_{i+1}|\ (i=1,\ldots,N-1). \tag{3.2}$$

According to a fundamental theorem of algebra,

$$P_N=\gamma_f\prod_{i=1}^{N}(\mu-\mu_i)=\gamma_f\sum_{k=0}^{N}(-1)^{N-k}\sigma_{N-k}(\mu)\,\mu^k, \tag{3.3}$$

where the roots $\mu_1, \ldots, \mu_N$ serve as the arguments of the sums $\sigma_{N-k}(\mu)$. A comparison of Eqs. (3.1) and (3.3) with $N-k=j$ gives

$$(-1)^j\sigma_j(\mu)=\frac{\gamma^{(N-j)}}{\gamma_f}\sigma_j. \tag{3.4}$$

Let us establish an asymptotic distribution of the roots of the characteristic equation on the assumption

$$\lambda_1\gg\lambda_2\gg\ldots\gg\lambda_{N-1}\gg\lambda_N. \tag{3.5}$$

Conditions of this kind very often characterize real chemical mixtures, where the reaction rates may differ by several orders of magnitude. In addition, we will assume in what follows that each of the ratios e_i^2/λ_i ($i=1, \ldots, N$) is comparable to unity in order of magnitude.

The inequalities (3.5) permit simplifying considerably the expression for the intermediate sound velocities. Using the results expounded in [10], one can show that Eq. (1.7) reduces to

$$a_{k\infty}=a_{f\infty}-\frac{1}{2}\delta_a^2 v_\infty\sum_{m=1}^{N-k}\frac{e_m^2}{\lambda_m}. \tag{3.6}$$

When $k=0$, the exact relation (1.10) between the frozen and equilibrium sound velocities follows. Returning to Eq. (1.6), we have

$$\gamma^{(k)}=\gamma_f+\frac{1}{2}\frac{p_\infty}{\rho_\infty v_\infty^2}\sum_{m=1}^{N-k}\frac{e_m^2}{\lambda_m}. \tag{3.7}$$

Let us consider the various cases which may be encountered in connection with the solution of the characteristic equation. First, let all the numbers $\gamma^{(k)}\sim 1$ ($k=0, \ldots, N$). Then a root should exist which is much larger than or of the order λ_1 in absolute magnitude. According to the numbering established by the inequalities (3.2), this is μ_1. Bearing in mind the conditions (3.5), we preserve only the main terms in Eq. (3.1). As a result, we find the approximate value

$$\mu_1=-\frac{\gamma^{(N-1)}}{\gamma_f}\lambda_1. \tag{3.8}$$

The next roots $\mu_2, \ldots, \mu_N$ are determined by the equation

$$P_{N-1}=\prod_{i=2}^{N}(\mu-\mu_i)=\sum_{k=0}^{N-1}(-1)^{N-1-k}\sigma_{N-1-k}^{(1)}(\mu)\,\mu^k=0,$$

where the superscript (1) next to the sum $\sigma^{(1)}(\mu)$ indicates that μ_1 is excluded from the complete set of its constituent numbers μ_i. We write down the identity

$$\sigma_j(\mu) = \mu_1\sigma_{j-1}^{(1)}(\mu) + \sigma_j^{(1)}\overline{(\mu)}. \tag{3.9}$$

It is possible to show that the estimate $|\mu_1| \gg |\mu_2|$ is valid for the root μ_2. Making use of Eqs. (3.4) and (3.8), we obtain approximately

$$\sigma_{N-1-k}^{(1)}(\mu) = (-1)^{N-1-k}\frac{\gamma^{(k)}}{\gamma^{(N-1)}}\frac{\sigma_{N-k}}{\lambda_1},$$

and then

$$P_{N-1} = \frac{1}{\lambda_1\gamma^{(N-1)}}\sum_{k=0}^{N-1}\sigma_{N-k}\gamma^{(k)}\mu^k.$$

The result of the application of the identity (3.9) to the sum σ_{N-k} reads $\sigma_{N-k} = \lambda_1\sigma_{N-1-k}^{(1)}$ if one neglects terms which are not dependent on λ_1. Finally,

$$P_{N-1} = \frac{1}{\gamma^{(N-1)}}\sum_{k=0}^{N-1}\sigma_{N-1-k}^{(1)}\gamma^{(k)}\mu^k. \tag{3.10}$$

A comparison of Eqs. (3.1) and (3.10) shows that the polynomials P_N and P_{N-1} are similar in their structure. From this it immediately follows that an equation of the type (3.8) is valid for the root μ_2.

Continuing the procedure described, one can calculate the roots $\mu_3, \ldots, \mu_N$. Asymptotic expressions for them are evident:

$$\mu_i = \frac{\gamma^{(N-i)}}{\gamma^{(N-i+1)}}\lambda_i \quad (i = 1, \ldots, N). \tag{3.11}$$

All these roots are negative with the exception of one. By virtue of the inequalities (1.8) and (2.1) and Eqs. (3.6) and (3.7), the asymptotic representation of the single positive root μ_j is distinguished by the conditions

$$\mu_i = -\frac{\gamma^{(N-j)}}{\gamma^{(N-j+1)}}\lambda_j, \quad \gamma^{(N-j+1)} < 0 < \gamma^{(N-j)}. \tag{3.12}$$

When $i = 1$, Eq. (3.11) changes into (3.8).

Let us consider the case in which $|\gamma^{(N-1)}| \ll 1$, and the remaining constants $\gamma^{(k)}$ are, in agreement with Eq. (3.7), comparable in order of magnitude with unity, $\gamma_f < 0$, and $\gamma^{(N-i)} > 0$ $(i = 2, \ldots, N)$. Let us set the index $j = 2$ in Eq. (3.4), after which we preserve only the main terms

$$\sigma_2(\mu) = \frac{\gamma^{(N-2)}}{\gamma_f}\lambda_1\lambda_2$$

in it. If all the roots $\mu_i (i = 1, \ldots, N)$ would be of the same order of magnitude as λ_2 or even smaller in absolute magnitude than this number, the last equation would not be satisfied. Among the roots of the characteristic equation there necessarily exist those whose absolute magnitude exceeds λ_2. For their determination we extract from (3.1) the approximate equation

$$\gamma_f\mu^2 + \lambda_1\gamma^{(N-1)}\mu + \lambda_1\lambda_2\gamma^{(N-2)} = 0.$$

Adhering to the numbering established by the requirements (3.2), we have

$$\mu_{1,2} = -\frac{1}{2}\lambda_1\frac{\gamma^{(N-1)}}{\gamma_f} \pm \sqrt{\frac{1}{4}\lambda_1^2\left[\frac{\gamma^{(N-1)}}{\gamma_f}\right]^2 - \lambda_1\lambda_2\frac{\gamma^{(N-2)}}{\gamma_f}} \tag{3.13}$$

with the ratio $\gamma^{(N-2)}/\gamma_f < 0$.

In order to find the next roots $\mu_3, \ldots, \mu_N$, it is necessary to solve the equation

$$P_{N-2} = \prod_{i=3}^{N}(\mu - \mu_i) = \sum_{k=0}^{N-2}(-1)^{N-2-k}\sigma_{N-2-k}^{(1,2)}(\mu)\mu^k = 0.$$

Let us use the identity

$$\sigma_j(\mu) = \mu_1\mu_2\sigma_{j-2}^{(1,2)}(\mu) + (\mu_1 + \mu_2)\sigma_{j-1}^{(1,2)}(\mu) + \sigma_j^{(1,2)}(\mu). \tag{3.14}$$

Since $|\mu_{1,2}| \gg |\mu_3|$, the result of the application of Eqs. (3.4) and (3.13) is written approximately in the form

$$\sigma_{N-2-k}^{(1,2)}(\mu) = (-1)^{N-k}\frac{\gamma^{(k)}}{\gamma^{(N-2)}}\frac{\sigma_{N-k}}{\lambda_1\lambda_2}.$$

The polynomial P_{N-2} is converted to the form

$$P_{N-2}=\frac{1}{\lambda_1\lambda_2\gamma^{(N-2)}}\sum_{k=0}^{N-2}\sigma_{N-k}\gamma^{(k)}\mu^k.$$

Returning to the identity (3.14) for the sum σ_{N-k}, we find that its asymptotic form is $\sigma_{N-k}=\lambda_1\lambda_2\sigma_{N-2-k}^{(1,2)}$ if one keeps only the main term in it. Finally,

$$P_{N-2}=\frac{1}{\gamma^{(N-2)}}\sum_{k=0}^{N-2}\sigma_{N-2-k}\gamma^{(k)}\mu^k. \tag{3.15}$$

It is evident from a comparison of Eq. (3.15) with (3.1) and (3.10) that the polynomial P_{N-2} is similar in its structure to the polynomials P_N and P_{N-1}. As is clear from this, the desired roots μ_3, ..., μ_N are specified by Eq. (3.11) with $i=3$, ..., N, and all of them are negative. The value of the single positive root is established by Eq. (3.13) with the upper sign in front of the radical on its right-hand side.

Finally, let the condition $|\gamma^{(N-j)}|\ll 1$ exist with $j=1$, ..., $N-1$. In accordance with the inequalities (1.8) and (2.1) and Eqs. (3.6) and (3.7), the remaining constants $\gamma^{(N-i)}<0$ for $i=0$, ..., $j-1$, whereas $\gamma^{(N-i)}>0$ for $i=j+1$, ..., N. In this case the first $j-1$ negative roots of the characteristic equation are evidently found from Eq. (3.11), in which $i=1$, ..., $j-1$. The next two roots are

$$\mu_{j,j+1}=-\frac{1}{2}\lambda_j\frac{\gamma^{(N-j)}}{\gamma^{(N-j+1)}}\pm\sqrt{\frac{1}{4}\lambda_j^2\left[\frac{\gamma^{(N-j)}}{\gamma^{(N-j+1)}}\right]^2-\lambda_j\lambda_{j+1}\frac{\gamma^{(N-j-1)}}{\gamma^{(N-j+1)}}}. \tag{3.16}$$

The positive one of them is obtained by selecting the upper sign in front of the radical on the right-hand side of (3.16). The remaining $N-j-1$ negative roots of the characteristic equation are determined by Eqs. (3.11), in which $i=j+2$, ..., N.

4. Quasiequilibrium Mode. Let us assume that the velocity of the advancing flow remains less even than the first intermediate velocity $\alpha_{1\infty}$, although it exceeds the equilibrium velocity $a_{e\infty}$. Then $\gamma^{(N-i)}<0$ when $i=0$, ..., $N-1$, and only $\gamma_e>0$. We introduce a new scale of length by means of

$$x=x_N/\lambda_N. \tag{4.1}$$

The first equation of the original system (1.1) takes the form

$$2\left(\varepsilon m_\infty v+\varepsilon_a^2\gamma_f\right)\frac{dv}{dx_N}=-\delta_a^2\sum_{i=1}^{N}\frac{1}{\lambda_N}e_i(\lambda_i q_i+e_i v), \tag{4.2}$$

whereas the remaining equations for the components of the completeness vector of the chemical reactions are of the form

$$\frac{dq_i}{dx_N}=-\frac{\lambda_i}{\lambda_N}\left(q_i+\frac{e_i}{\lambda_i}v\right)\quad(i=1,\ldots,N-1),$$
$$\frac{dq_N}{dx_N}=-\left(q_N+\frac{e_N}{\lambda_N}v\right). \tag{4.3}$$

Let us eliminate the thermodynamic variable q_1 from the system of equations (4.2) and (4.3). Finally, we obtain

$$\frac{\lambda_N}{\lambda_1}\frac{d}{dx_N}\left[(\varepsilon m_\infty v+\varepsilon_a^2\gamma_f)\frac{dv}{dx_N}\right]+\frac{1}{2}\delta_a^2\sum_{i=2}^{N}e_i\left(\frac{\lambda_i}{\lambda_1}-1\right)\frac{dq_i}{dx_N}+\left[\varepsilon m_\infty v+\varepsilon_a^2\gamma_f+\frac{1}{2}\delta_a^2\left(\frac{e_1^2}{\lambda_1}+\sum_{i=2}^{N}\frac{e_i^2}{\lambda_i}\frac{\lambda_i}{\lambda_1}\right)\right]\frac{dv}{dx_N}=0.$$

Here in accordance with the conditions (3.5) we proceed to the limit as $\lambda_i/\lambda_1\to 0$ ($i=2$, ..., N). Taking account of the fact that the ratio $e_i^2/\lambda_i\sim 1$ for any $i=1$, ..., N, and making use of Eq. (3.7) with $k=N-1$, we have

$$2\left(\varepsilon m_\infty v+\varepsilon_a^2\gamma^{(N-1)}\right)\frac{dv}{dx_N}=-\delta_a^2\sum_{i=2}^{N}\frac{1}{\lambda_N}e_i(\lambda_i q_i+e_i v). \tag{4.4}$$

The above equation agrees with the original Eq. (4.2) if one replaces in the latter the constant γ_f by $\gamma^{(N-1)}$, and, in addition, sums over the index i not from 1 to N but from 2 to N. We eliminate the second thermodynamic constant q_2 from the system in question. As a result, an equation of the type (4.4) is obtained in which $\gamma^{(N-2)}$ replaces the parameter $\gamma^{(N-1)}$ and the index i runs through the values 3, ..., N.

We continue the indicated procedure, accompanying the elimination of the next thermodynamic variable q_j each time by the limiting transition of $\lambda_i/\lambda_j\to 0$ ($i=j+1$, ..., N). After the $(N-1)$-th repetition of this procedure a comparatively simple system of equations arises,

$$2\left(\varepsilon m_\infty v + \varepsilon_a^2\gamma^{(1)}\right)\frac{dv}{dx_N} = \delta_a^2 e_N \frac{dq_N}{dx_N},$$
$$\frac{dq_N}{dx_N} = -\left(q_N + \frac{e_N}{\lambda_N}v\right), \tag{4.5}$$

which contains only the two desired functions v and q_N. If the solution of (4.5) is known, the thermodynamic variables $q_1, \ldots, q_{N-1}$ are determined from the final relationships

$$q_i = -\frac{e_i}{\lambda_i}v \quad (i = 1, \ldots, N-1). \tag{4.6}$$

Their meaning is exceedingly simple: They are the conditions for the equilibrium occurrence of the first $N-1$ reactions. The process of gas compression is concluded after the N-th reaction arrives at a new equilibrium state.

We will formulate the initial conditions which should be satisfied when the system (4.5) is integrated. We obtain

$$v = c\,e^{\mu_N x}, \quad q_i = \frac{e_i c}{\lambda_i}e^{\mu_N x} \quad (i = 1, \ldots, N-1),$$
$$q_N = \frac{e_N c}{\lambda_N}\frac{\gamma^{(1)}}{\gamma^{(0)} - \gamma^{(1)}}e^{\mu_N x}, \qquad \mu_N = -\frac{\gamma^{(0)}}{\gamma^{(1)}}\lambda_N \tag{4.7}$$

from the asymptotic representations (2.9) with $\lambda_N/\lambda_i \to 0$ ($i = 1, \ldots, N-1$) as $x_N \to -\infty$. These equations show that the asymptotic values of the functions v and q_i($i = 1, \ldots, N-1$) correspond to the final relationships (4.6).

The system (4.5) is equivalent to the single second-order equation

$$\frac{d}{dx_N}\left[\left(\varepsilon m_\infty v + \varepsilon_a^2\gamma^{(1)}\right)\frac{dv}{dx_N}\right] + \left(\varepsilon m_\infty v + \varepsilon_a^2\gamma_e\right)\frac{dv}{dx_N} = 0. \tag{4.8}$$

It is also possible to obtain the latter equation from Eq. (1.5) by substituting the variable (4.1) into it and performing the limiting transition of $\lambda_{i+1}/\lambda_i \to 0$ ($i = 1, \ldots, N-1$). The integral of (4.8) satisfying the asymptotic initial data (4.7) is of the form

$$c_N - x_N = \frac{\gamma^{(1)}}{\gamma^{(0)}}\ln|v| + \left(2 - \frac{\gamma^{(1)}}{\gamma^{(0)}}\right)\ln\left|\varepsilon_a^2\gamma^{(0)} + \frac{1}{2}\varepsilon m_\infty v\right|. \tag{4.9}$$

It permits a significant simplification when $|\gamma^{(0)}| \ll 1$. In this case we introduce a new function being sought:

$$u = \frac{\varepsilon_a^2}{\varepsilon m_\infty}\gamma^{(0)} + v,$$

and denote the constant quantities by

$$\sigma = \frac{\varepsilon_a^2}{\varepsilon m_\infty}\gamma^{(0)}, \quad l = -\frac{2\varepsilon_a^2}{\varepsilon m_\infty}\gamma^{(1)}, \quad \exp\left[\frac{\gamma^{(0)}}{\gamma^{(1)}}d_N\right] = \frac{\varepsilon m_\infty}{2}\exp\left[\frac{\gamma^{(0)}}{\gamma^{(1)}}c_N\right].$$

Then

$$u = -\sigma\,\mathrm{th}\,\frac{\sigma}{l}(x_N - d_N).$$

This is the well-known Taylor solution, which describes the structure of a shock wave in a viscous heat-conducting gas [11]. As the amplitude tends to zero, the quasiequilibrium process of compression of the relaxing mixture in which a single reaction occurs conforms to this solution [5, 6, 12]. As the discussions presented above show, the Taylor solution also specifies the structure of a weak shock wave in a chemically active mixture with an arbitrary number of reactions on the condition that the velocity v_∞ of the advancing flow exceeds only slightly the equilibrium sound velocity $a_{e\infty}$. As the flow velocity at infinity increases in the range $a_{e\infty} < v_\infty < \alpha_{1\infty}$, the compression of the mixture inside the single relaxation zone is determined by Eq. (4.9).

5. General Case. The subsequent increase in the velocity of the advancing flow leads to the necessity of considering the situation in which this velocity falls within the interval $a_{e\infty} < v_\infty < a_{f\infty}$. For example, let $\alpha_{N-j,\infty} < v_\infty < \alpha_{N-j+1,\infty}$; then $\gamma^{(N-j+1)} < 0 < \gamma^{(N-j)}$. We will assume that both parameters $\gamma^{(N-j)}$ and $\gamma^{(N-j+1)}$ are comparable to unity in order of magnitude.

Taking Eq. (3.12) for the single positive root of the characteristic equation into account, we introduce the variable

$$x = x_j/\lambda_j. \tag{5.1}$$

The first equation of the original system (1.1) takes the form

$$2\left(\varepsilon m_\infty v + \varepsilon_a^2 \gamma_j\right) \frac{dv}{dx_j} = -\delta_a^2 \sum_{i=1}^{N} \frac{1}{\lambda_j} e_i \left(\lambda_i q_i + e_i v\right). \tag{5.2}$$

The remaining equations give

$$\frac{dq_i}{dx_j} = -\frac{\lambda_i}{\lambda_j}\left(q_i + \frac{e_i}{\lambda_i} v\right) \quad (i = 1, \ldots, j-1), \tag{5.3}$$

$$\frac{dq_j}{dx_j} = -\left(q_j + \frac{e_j}{\lambda_j} v\right), \quad \frac{dq_k}{dx_j} = 0 \quad (k = j+1, \ldots, N)$$

with the requirements $\lambda_k/\lambda_j \to 0$ ($k = j+1, \ldots, N$) taken into account.

One can eliminate the thermodynamic variable q_1 from the system of Eqs. (5.2) and (5.3). After the limiting transition of $\lambda_i/\lambda_1 \to 0$ ($i = 2, \ldots, j$) Eq. (4.4), in which the summation over the index i on the right-hand side is taken from 2 to j, is evidently obtained. We continue the procedure of eliminating thermodynamic variables according to the rule described in the preceding section. As a result of the (j−1)-th repetition of this procedure we arrive at the system of equations

$$2\left(\varepsilon m_\infty v + \varepsilon_a^2 \gamma^{(N-j+1)}\right) \frac{dv}{dx_j} = \delta_a^2 e_j \frac{dq_j}{dx_j}, \tag{5.4}$$

$$\frac{dq_j}{dx_j} = -\left(q_j + \frac{e_j}{\lambda_j} v\right), \quad \frac{dq_k}{dx_j} = 0 \quad (k = j+1, \ldots, N).$$

After its solution is constructed, the first $j-1$ thermodynamic variables $q_1, \ldots, q_{j-1}$ are calculated from the final Eqs. (4.6). The situation which has arisen can be interpreted naturally: The first $j-1$ reactions proceed uniformly, and the last $N-j$ reactions are in a frozen state.

Now we will investigate what initial conditions must be imposed in connection with the integration of the system (5.4). It follows from the asymptotic representations (2.9) with $\lambda_j/\lambda_i \to 0$ ($i = 1, \ldots, j-1$) that as $x_j \to -\infty$

$$v = c\, e^{\mu_j x}, \quad q_i = -\frac{e_i c}{\lambda_i} e^{\mu_j x}, \quad i = 1, \ldots, j-1, \tag{5.5}$$

$$q_j = \frac{e_j c}{\lambda_j} \frac{\gamma^{(N-j+1)}}{\gamma^{(N-j)} - \gamma^{(N-j+1)}} e^{\mu_j x}, \quad q_k = 0, \quad k = j+1, \ldots, N,$$

$$\mu_j = -\frac{\gamma^{(N-j)}}{\gamma^{(N-j+1)}} \lambda_j.$$

The above equations show that the asymptotic values of the functions v and q_i ($i = 1, \ldots, j-1$) are in agreement with the final Eqs. (4.6). As is evident from the differential equations and the initial data for the variables q_k ($k = j+1, \ldots, N$), these thermodynamic variables remain equal to zero in the relaxation zone in question.

System (5.4) is equivalent to the single second-order equation

$$\frac{d}{dx_j}\left[\left(\varepsilon m_\infty v + \varepsilon_a^2 \gamma^{(N-j+1)}\right) \frac{dv}{dx_j}\right] + \left(\varepsilon m_\infty v + \varepsilon_a^2 \gamma^{(N-j)}\right) \frac{dv}{dx_j} = 0, \tag{5.6}$$

which coincides with (4.8) when $j = N$. Of course, the above equation follows from (1.5) if one uses the variable (5.1) and performs the limiting process for all $\lambda_{i+1}/\lambda_i \to 0$ ($i = 1, \ldots, N-1$). The integral of Eq. (5.6) with the asymptote (5.5) has the form

$$c_j - x_j = \frac{\gamma^{(N-j+1)}}{\gamma^{(N-j)}} \ln|v| + \left(2 - \frac{\gamma^{(N-j+1)}}{\gamma^{(N-j)}}\right) \ln\left|\varepsilon_a^2 \gamma^{(N-j)} + \frac{1}{2} \varepsilon m_\infty v\right| \tag{5.7}$$

as $x_j \to -\infty$ and changes into (4.8) when $j = N$.

A large difference exists between the gas motion under discussion and that which was investigated in Sec. 4. When $a_{e\infty} < v_\infty < \alpha_{1\infty}$, the compression process concludes with the arrival at equilibrium of the N-th reaction in a single relaxation zone. In the general case $\alpha_{N-j,\infty} < v_\infty < \alpha_{N-j+1,\infty}$ the tending of the j-th reaction to equilibrium does not complete the compression of the chemically active mixture. Stretching behind the first relaxation zone are other wider relaxation regions in which the dominant role belongs to reactions with the numbers $j+1, \ldots, N$.

In order to construct the perturbation field in the next relaxation zone, we introduce the asymptotic expression of the integral (5.7) as $x_j \to +\infty$

$$v = -\frac{2\varepsilon_a^2}{\varepsilon m_\infty}\gamma^{(N-j)} + b_j \exp\left[-\frac{\gamma^{(N-j)}}{2\gamma^{(N-j)} - \gamma^{(N-j+1)}} x_j\right], \tag{5.8}$$

$$b_j = \frac{2}{\varepsilon m_\infty}\left[\frac{2\varepsilon_a^2}{\varepsilon m_\infty}\gamma^{(N-j)}\right]^{-\frac{\gamma^{(N-j+1)}}{2\gamma^{(N-j)}-\gamma^{(N-j+1)}}} \exp\left[\frac{\gamma^{(N-j)} c_j}{2\gamma^{(N-j)} - \gamma^{(N-j+1)}}\right],$$

which remains valid when $j = N$. From the second equation of the system (5.4) we obtain the asymptote of the thermodynamic variable

$$q_j = \frac{e_j}{\lambda_j}\left\{\frac{2\varepsilon_a^2}{\varepsilon m_\infty}\gamma^{(N-j)} + \frac{2\gamma^{(N-j)} - \gamma^{(N-j+1)}}{\gamma^{(N-j)}} b_j \exp\left[-\frac{\gamma^{(N-j)} x_j}{2\gamma^{(N-j)} - \gamma^{(N-j+1)}}\right]\right\} + d_j \exp(-x_j)$$

with the new variable d_j. In other words, at the end of the first relaxation zone the thermodynamic variable in question tends to its own equilibrium value

$$q_j = -\frac{e_j}{\lambda_j} v_{sj}, \quad v_{sj} = -\frac{2\varepsilon_a^2}{\varepsilon m_\infty}\gamma^{(N-j)}. \tag{5.9}$$

If one calculates the parameters of the particles behind the shock front (discontinuity) in a mixture in which the first $a_{f\infty}$ reactions have reached equilibrium, the role of the frozen sound velocity $a_{f\infty}$ will evidently be played by the intermediate velocity $\alpha_{N-j,\infty}$. The quantity v_{sj} is nothing else but the velocity of the gas particles in the case of such a sudden compression.

As is evident directly from the original Eqs. (1.1), the scale of the next relaxation zone should be specified by means of $x = x_{j+1}/\lambda_{j+1}$ under conditions of the equilibrium occurrence of the first j reactions. Discussions which are completely analogous to those above lead to the system of equations (5.4) with the replacement in it of the subscript j by j + 1. After its solution the thermodynamic variables $q_1, \ldots, q_j$ can be calculated with the help of the final Eqs. (4.6).

In order to supply the initial data which are necessary for integration of the differential equations in the second relaxation zone, we make use of the principle of splicing the exterior and interior asymptotic expansions [13]. Thus, as

$$x_{j+1} = \frac{\lambda_{j+1}}{\lambda_j} x_j,$$

we find from the limiting conditions (5.9) at $x_{j+1} = 0$

$$v = -\frac{2\varepsilon_a^2}{\varepsilon m_\infty}\gamma^{(N-j)}, \quad q_i = \frac{2\varepsilon_a^2}{\varepsilon m_\infty}\frac{e_i}{\lambda_i}\gamma^{(N-j)} \quad (i = 1, \ldots, j), \tag{5.10}$$
$$q_k = 0 \quad (k = j+1, \ldots, N).$$

The corresponding initial values of the derivatives are

$$\frac{dv}{dx_{j+1}} = -\frac{\delta_a^2}{\varepsilon m_\infty}\frac{e_{j+1}^2}{\lambda_{j+1}}, \quad \frac{dq_{j+1}}{dx_{j+1}} = \frac{2\varepsilon_a^2}{\varepsilon m_\infty}\frac{e_{j+1}}{\lambda_{j+1}}\gamma^{(N-j)}. \tag{5.11}$$

In order to construct the solution of the Cauchy problem which has been formulated, we apply the second-order Eq. (5.6), having in advance replaced the subscript j in it by j + 1. Its integration gives

$$\left(\varepsilon m_\infty v + \varepsilon_a^2\gamma^{(N-j)}\right)\frac{dv}{dx_{j+1}} + \frac{1}{2}\varepsilon m_\infty v^2 + \varepsilon_a^2\gamma^{(N-j-1)}v = c_{j+1}.$$

Substituting the initial data (5.10) and (5.11) here, we find that the arbitrary constant $C_{j+1} = 0$. This value of the constant C_{j+1} will lead to the integral (5.7), in which j + 1 appears instead of j. Eqs. (5.8) determine the asymptote of the perturbations upon exiting from the second relaxation zone, i.e., as $x_{j+1} \to \infty$.

This process can be continued without any changes at all. The solution obtained as a result gives an asymptotic description of the motion of a chemically active mixture inside a completely dispersed shock wave. When $\alpha_{N-j,\infty} < v_\infty < \alpha_{N-j+1,\infty}$, the perturbed region is split into N − j + 1 relaxation zones, in each of which the dominant role belongs to a single reaction. The width of all the preceding zones tends to zero on the scale of any succeeding one. Therefore, continuous compression of the gas in any relaxation zone turns out to be equivalent to its sudden compression in a shock front. It is possible in the investigation of the last stage of the process in the (N − j + 1)-th zone to treat the first N − j relaxation zones as a sequence of N − j discontinuities.

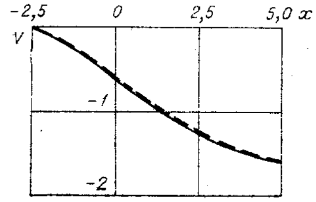

Fig. 1

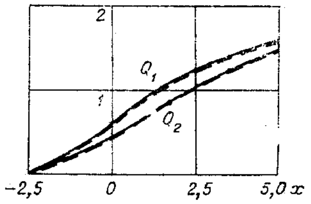

Fig. 2

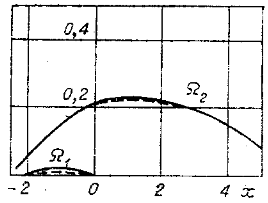

Fig. 3

To illustrate the theory developed, calculations have been made of the structure of a completely dispersed shock wave in a mixture in which two reactions are occurring. It was assumed in the calculations that $\varepsilon_a^2/\varepsilon m_\infty = 1$, $\delta^2/\varepsilon_a^2 = 2$, $\lambda_1 = 100$, $e_1 = 10$, $\lambda_2 = 1$, $e_2 = 1$, and $\gamma_f = -1.5$. The results are illustrated in Figs. 1-3, in which the solid lines correspond to the exact numerical solution of the problem, and the dashed lines correspond to the data of the asymptotic analysis. The variables $V = v/\gamma_e$, $Q_1 = \lambda_1 q_1/(\gamma_e e_1)$, $Q_2 = \lambda_2 q_2/(\gamma_e e_2)$, $\Omega_1 = \omega_1/(\gamma_e e_1)$, and $\Omega_2 = \omega_2/(\gamma_e e_2)$ were used in plotting Figs. 1-3, and the coordinate $x_2 = \lambda_2 x = x$ was selected as the independent variable. The results of the asymptotic analysis agree well with those resulting from the numerical solution of the original system (1.1).

6. Transition through an Intermediate Sound Velocity. We will investigate what happens when the flow velocity v_∞ is close to some intermediate velocity $\alpha_{N-j,\infty}$. In this transition case $|\gamma^{(N-j)}| \ll 1$. In accordance with Eq. (3.16) we will make the transformation

$$x = x_j/\mu_j \tag{6.1}$$

and we will understand by μ_j the single positive root of the characteristic equation. Transforming from the original system of equations (1.1) according to the rule expounded in the two preceding sections, we have

$$2\left(\varepsilon m_\infty v + \varepsilon_a^2\gamma^{(N-j+1)}\right)\frac{dv}{dx_j} = \delta_a^2\left(e_j\frac{dq_j}{dx_j} + e_{j+1}\frac{dq_{j+1}}{dx_j}\right),$$
$$\frac{dq_j}{dx_j} = -\frac{\lambda_j}{\mu_j}\left(q_j + \frac{e_j}{\lambda_j}v\right),\quad \frac{dq_{j+1}}{dx_j} = -\frac{\lambda_{j+1}}{\mu_j}\left(q_{j+1} + \frac{e_{j+1}}{\lambda_{j+1}}v\right),$$
$$\frac{dq_k}{dx_j} = 0,\quad k = j+2,\ \ldots,\ N. \tag{6.2}$$

After solution of the system of equations (6.2) the first $j-1$ thermodynamic variables $q_1, \ldots, q_{j-1}$ are recovered with the help of the final Eqs. (4.6). The relaxation process is characterized by the fact that it is impossible to treat the j-th and (j+1)-th reactions independently. Although their rates differ in order of magnitude, precisely the mutual effect of both reactions determines the structure of the perturbation field as $x \to -\infty$. The introduction of the scale (6.1) is dictated by this fact.

The initial conditions for the system (6.2) as $x_j \to -\infty$ are of the form

$$v = ce^{\mu_j x},\quad q_i = -\frac{e_i c}{\lambda_i}e^{\mu_j x}\quad (i = 1,\ \ldots,\ j-1),$$
$$q_j = -\frac{e_j c}{\lambda_j + \mu_j}e^{\mu_j x},\quad q_{j+1} = -\frac{e_{j+1}c}{\lambda_{j+1} + \mu_j}e^{\mu_j x},$$
$$q_k = 0\quad (k = j+2,\ \ldots,\ N). \tag{6.3}$$

It is clear that the asymptotic values of the functions v and $q_i (i = 1, \ldots, j-1)$ correspond to the final Eqs. (4.6).

The system (6.2) is equivalent to the single third-order equation

$$\frac{d^2}{dx_j^2}\left[\left(\varepsilon m_\infty v + \varepsilon_a^2\gamma^{(N-j+1)}\right)\frac{dv}{dx_j}\right] + \frac{\lambda_j}{\mu_j}\frac{d}{dx_j}\left[\left(\varepsilon m_\infty v + \varepsilon_a^2\gamma^{(N-j)}\right)\frac{dv}{dx_j}\right] + \frac{\lambda_j\lambda_{j+1}}{\mu_j^2}\left(\varepsilon m_\infty v + \varepsilon_a^2\gamma^{(N-j-1)}\right)\frac{dv}{dx_j} = 0, \tag{6.4}$$

which can also be obtained directly from Eq. (1.5) if one converts to the variable (6.1) in Eq. (1.5). Integration of Eq. (6.4) with the initial data (6.3) taken into account gives

$$\frac{d}{dx_j}\left[\left(\varepsilon m_\infty v + \varepsilon_a^2\gamma^{(N-j+1)}\right)\frac{dv}{dx_j}\right] + \frac{\lambda_j}{\mu_j}\left(\varepsilon m_\infty v + \varepsilon_a^2\gamma^{(N-j)}\right)\frac{dv}{dx_j} + \frac{\lambda_j\lambda_{j+1}}{\mu_j^2}\left(\frac{1}{2}\varepsilon m_\infty v^2 + \varepsilon_a^2\gamma^{(N-j-1)}v\right) = 0.$$

From this it is possible to establish the asymptotic representation

$$v = -\frac{2\varepsilon_a^2}{\varepsilon m_\infty}\gamma^{(N-j-1)} + b_{j+1}\exp\left[-\frac{\gamma^{(N-j-1)}}{2\gamma^{(N-j-1)} - \gamma^{(N-j)}}\frac{\lambda_{j+1}}{\mu_j}x_j\right]$$

for the function v, which is valid as $x_j \to +\infty$. Thus at the end of the first relaxation zone the thermodynamic variables q_j and q_{j+1} tend to the equilibrium values

$$q_j = -\frac{e_j}{\lambda_j}v_{s,j+1}, \quad q_{j+1} = -\frac{e_{j+1}}{\lambda_{j+1}}v_{s,j+1}, \quad v_{s,j+1} = -\frac{2\varepsilon_a^2}{\varepsilon m_\infty}\gamma^{(N-j-1)}. \tag{6.5}$$

Eqs. (6.5) are analogous to (5.9); therefore, construction of the solution in the remaining $N-j-1$ relaxation zones follows the method indicated in Sec. 5.

We note the limiting case $\gamma^{(N)}=0$. Right up to some point $x=x^*$ the flow remains unperturbed, i.e., $v=q_i=0$, $i=1, \ldots, N$. Let $x^*=0$; then the solution has the form [12]

$$v = -\frac{2\varepsilon_a^2}{\varepsilon m_\infty}\gamma^{(N-1)}\left[1-\exp\left(-\frac{1}{2}\lambda_1 x\right)\right]$$

in the first relaxation zone. At the point $x=0$ the parameters of the gas are continuous, but their derivatives with respect to the spatial coordinate vary discontinuously. Evidently, this point corresponds to the characteristic of the differential equations which the flows of relaxing mixtures satisfy. Continuation of the solution to the remaining $N-1$ relaxation zones is accomplished according to the standard procedure described above.

LITERATURE CITED

1. A. L. Ni and O. S. Ryzhov, "The equations of transonic flows of relaxing mixtures," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 4 (1977).
2. L. G. Napolitano, "Generalized velocity potential equation for pluri-reacting mixtures," Arch. Mech. Stosowanej, 16, No. 2 (1964).
3. L. G. Napolitano, "Nonlinear nonequilibrium flows," I. A. Rept. No. 142 (1969).
4. A. L. Ni and O. S. Ryzhov, "Sound velocities in multicomponent chemically active gas mixtures," Vestn. Leningr. Univ. Mat. Mekh. Astron., No. 13, Issue 3 (1976).
5. E. V. Stupochenko, S. A. Losev, and A. I. Osipov, Relaxation Processes in Shock Waves [in Russian], Nauka, Moscow (1965).
6. J. Clarke and M. McChesney, The Dynamics of Real Gases [Russian translation], Mir, Moscow (1967).
7. J. F. Clarke and J. B. Rodgers, "Shock waves in a gas with several relaxing internal energy modes," J. Fluid Mech., 21, Part 4 (1965).
8. A. L. Ni, "The propagation of weak shock waves in media with an arbitrary number of chemical reactions," Prikl. Mat. Mekh., 41, No. 6 (1977).
9. I. M. Gel'fand, Lectures on Linear Algebra [in Russian], Nauka, Moscow (1971).
10. A. L. Ni and O. S. Ryzhov, "Limiting expressions for the intermediate sound velocities in nonequilibrium flows with an arbitrary number of chemical reactions," Prikl. Mat. Mekh., 41, No. 1 (1977).
11. G. I. Taylor, "The conditions necessary for discontinuous motion in gases," Proc. R. Soc., Ser. A, 84, No. 571 (1910).
12. O. S. Ryzhov, "The strictly transonic mode in flows of a reacting mixture," in: Problems of Applied Mathematics and Mechanics [in Russian], Nauka, Moscow (1971).
13. M. Van Dyke, Perturbation Methods in Fluid Mechanics, Academic (1964).